Two-dimensional fractional supersymmetric conformal and logarithmic conformal field theories and two point functions

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Abstract. A general two-dimensional fractional supersymmetric conformal field theory is investigated. The structure of the symmetries of the theory is studied. Applying the generators of the closed subalgebra generated by $(L_{-1}, L_0, G_{-1/(F+1)})$ and $(\bar{L}_{-1}, \bar{L}_0, \bar{G}_{-1/(F+1)})$, the two point functions of the component fields of supermultiplets are calculated. Then the logarithmic superconformal field theories are investigated and the chiral and full two point functions are obtained.

1 Introduction

2D conformally invariant field theories have become the subject of intense investigation in recent years, after the work of Belavin, Polyakov, and Zamolodchikov [1]. One of the main reasons for this is that 2D conformal field theories describe the critical behavior of two-dimensional statistical models [2–5]. Conformal field theory provides us with a simple and powerful means of calculating the critical exponents, as well as the correlation functions of the theory at the critical point [1,6]. Another application of conformal field theories is in string theories. Originally, string theory was formulated in flat 26-dimensional space-time for bosonic and flat 10-dimensional space-time for supersymmetric theories. It has been realized now that the central part of string theory is a 2D conformally invariant field theory. It is seen that the tree-level string amplitudes may be expressed in terms of the correlation functions of the corresponding conformal field theory on the plane, whereas string loop amplitudes may be expressed in terms of the correlation functions of the same conformal field theory on higher-genus Riemann surfaces [7–10].

Supersymmetry is a Z_2 extension of the Poincaré algebra [11,12]. But this can be enlarged to a superconformal algebra (see [13] for example). If the dimension of the space-time is two, there are also fractional supersymmetric extensions of the Poincaré and conformal algebra [14–17]. Fractional supersymmetry is a Z_n extension of the Poincaré algebra. In this paper, the general form of the two point functions of a theory with this symmetry is obtained.

According to Gurarie [18], conformal field theories with logarithmic correlation functions may be consistently defined. In some interesting physical theories like gravitational dressing of 2D field theories [19,20], polymers [21], WZNW models [22–25], percolation [26], the Haldane–Rezayi quantum Hall state [27], and edge excitation in fractional quantum Hall effect [28], there appear logarithmic correlation functions. Logarithmic operators are also seen in 2D magnetohydrodynamic turbulence [29–31], 2D turbulence [32,33] and some critical disordered models [34,35]. Logarithmic conformal field theories for the D-dimensional case ($D > 2$) have also been studied [36]. The two point functions of logarithmic conformal field theories are obtained in [34]. The general form of the correlators of the 2D logarithmic conformal field theories and logarithmic superconformal conformal field theories is investigated in [37,38] and [39], respectively.

In this paper, the general case $n = F + 1$ is considered. So the components of the superfield have grades $0, 1, ...,$ and F. The complex plane is extended by introducing two independent para-Grassmann variables θ and $\bar{\theta}$, satisfying $\theta^{F+1} = \bar{\theta}^{F+1} = 0$. One can develop an algebra, the fractional $n = F + 1$ algebra, based on these variables and the derivatives with respect to them [40–42]. In [43], the fractional supersymmetry has been investigated by introducing a certain fractional superconformal action. We do not consider any special action here. What we do is to use only the structure of the fractional superconformal symmetry to obtain general restrictions on the form of the two point functions. The scheme of this paper is the following. In Sect. 2, infinitesimal superconformal transformations are defined. In Sect. 3, the generators of these transformation and their algebra are investigated. In Sect. 4, the two point functions of such theories are obtained. In fact, Sects. 2–

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4 are a generalization of [44]. In Sect. 5, logarithmic superconformal field theories are investigated and the chiral and full two point functions of primary and quasi-primary (logarithmic) fields are obtained. This is a generalization of [45].

2 Infinitesimal superconformal transformations

Consider a para-Grassmann variable θ, satisfying

$$\theta^{F+1} = 0, \tag{1}$$

where $F \geq 2$ is a positive integer. A function of a complex variable z and this para-Grassmann variable will be of the form

$$f(z,\theta) = f_0(z) + \theta f_1(z) + \theta^2 f_2(z) + \ldots + \theta^F f_F(z). \tag{2}$$

The covariant derivative is defined as [16, 40, 46, 47]

$$D := \partial_\theta + \frac{q^F \theta^F}{(F)_{q^{-1}}!} \partial_z, \tag{3}$$

where

$$(F)_q! := (F)_q (F-1)_q \ldots (1)_q, \quad (m)_q = \frac{1-q^m}{1-q}, \tag{4}$$

and

$$\partial_\theta \theta = 1 + q\theta\partial_\theta. \tag{5}$$

q is an $F+1$th root of unity, with the property that there exists no positive integer m less than $F+1$ so that $q^m = 1$.

An infinitesimal transformation

$$z' = z + \sum_{k=0}^{F} \theta^k \omega_k(z),$$

$$\theta' = \theta + \sum_{k=0}^{F} \theta^k \epsilon_k(z), \tag{6}$$

is called superconformal if

$$D = (D\theta')D', \tag{7}$$

where

$$D' = \partial_{\theta'} + \frac{q^F \theta'^F}{(F)_{q^{-1}}!} \partial_{z'}; \tag{8}$$

from these, it is found that an infinitesimal superconformal transformation is of the form

$$z' = z + \omega_0(z) + \frac{q^F \theta^F}{(F)_{q^{-1}}!} \epsilon_0(z),$$

$$\theta' = \theta + \epsilon_0(z) + \frac{1}{F+1}\theta\omega_0'(z) + \sum_{k=2}^{F} \theta^k \epsilon_k(z), \tag{9}$$

where $\omega_0'(z) := \partial_z \omega_0(z)$. It is also seen that the following commutation relations hold:

$$\epsilon_i \theta = q\theta\epsilon_i, \quad i \neq 1. \tag{10}$$

One can extend these naturally to functions of z and $\bar{z}$, and θ and $\bar{\theta}$ (full functions instead of chiral ones). It is sufficient to define a covariant derivative for the pair $(\bar{z}, \bar{\theta})$, the analogue of (3), and extend the transformations (5), so that there are similar transformations for $(\bar{z}, \bar{\theta})$ as well. Then, defining a superconformal transformation as one satisfying (6) and its analogue for $(\bar{z}, \bar{\theta})$, one obtains, in addition to (7) and (8), similar expressions where $(z, \theta, \omega_k, \epsilon_k)$ are simply replaced by $(\bar{z}, \bar{\theta}, \bar{\omega}_k, \bar{\epsilon}_k)$. So, the superconformal transformations consist of two distinct class of transformations, the holomorphic and the anti-holomorphic, that do not communicate.

3 Generators of superconformal field theory

The (chiral) superfield $\phi(z, \theta)$, with the expansion,

$$\phi(z,\theta) = \varphi_0(z) + \theta\varphi_1(z) + \theta^2 \varphi_2(z) + \cdots + \theta^F \varphi_F(z), \tag{11}$$

is a superprimary field of the weight Δ if it transforms under a superconformal transformation as

$$\phi(z,\theta) \mapsto (D\theta')^{(F+1)\Delta} \phi(z',\theta'). \tag{12}$$

One can write this as

$$\phi(z,\theta) \mapsto \left[1 + \tilde{T}(\omega_0) + \tilde{S}(\epsilon_0) + \sum_{k=2}^{F} \tilde{H}_k(\epsilon_k)\right]\phi(z',\theta'), \tag{13}$$

to arrive at [40]

$$\tilde{T}(\omega_0) = \omega_0 \partial_z + \left(\Delta + \frac{\Lambda}{F+1}\right)\omega_0',$$

$$\tilde{S}(\epsilon_0) = \epsilon_0 \left[\delta_\theta + \frac{\theta^F}{(F)_{q^{-1}}!}\partial_z\right]$$

$$+ \frac{F+1}{(F)_{q^{-1}}}\Delta\epsilon_0'\theta^F,$$

$$\tilde{H}_k(\epsilon_k) = \theta^k \epsilon_k \delta_\theta. \tag{14}$$

Here Λ and δ_θ, are operators satisfying

$$[\Lambda, \theta] = \theta, \quad [\Lambda, \delta_\theta] = -\delta_\theta, \tag{15}$$

and

$$\delta_\theta \theta = q^F \theta \delta_\theta + 1. \tag{16}$$

One can now define the classical generators

$$l_n := \tilde{T}(z^{n+1}),$$

$$g_r := \tilde{S}(z^{r+1/(F+1)}), \tag{17}$$

where n and $r + 1/(F+1)$ are integers. We do not consider the generators corresponding to $\tilde{H}$, since there is no closed subalgebra, with a trivial central extension, containing these generators [16, 48]. The quantum generators of superconformal transformations are defined through

$$[L_n, \phi(z,\theta)] := l_n \phi,$$

$$[G_r, \phi(z,\theta)] := g_r \phi. \tag{18}$$

One can check that, apart from a possible central extension, these generator satisfy the following relations:

$$[L_n, L_m] = (n - m)L_{n+m}, \tag{19}$$

$$[L_n, G_r] = \left(\frac{n}{F+1} - r\right)G_{n+r}, \tag{20}$$

and

$$\{G_{r_0}G_{r_1}...G_{r_F}\}_{\text{per}} = (F+1)!L_{\left(\sum_{k=0}^{F} r_k\right)}, \tag{21}$$

where $\{...\}_{\text{per}}$ means the sum of products of all possible permutations of the generators G_{r_k}. This algebra has non-trivial central extensions; it has been shown [16,48] that there is only one subalgebra (containing G_r as well as L_n), the central extension for which is trivial. This algebra is the one generated by $[L_{-1}, L_0, G_{-1/(F+1)}]$ and their antiholomorphic counterparts $[\bar{L}_{-1}, \bar{L}_0, \bar{G}_{-1/(F+1)}]$. Having the effect of the L_n and G_r on the superfield, it is not difficult to obtain their effect on the component fields. For the L_n, the first equation of (14) leads directly to

$$[L_n, \varphi_k] = z^{n+1}\partial_z\varphi_k + (n+1)z^n\left(\Delta + \frac{k}{F+1}\right)\varphi_k. \tag{22}$$

This shows that the component field φ_k, is simply a primary field with the weight $\Delta + (k/(F+1))$. One can write (17), also in terms of the operator-product expansion:

$$\Re[T(\omega)\varphi_k(z)] \sim \frac{\partial_z\varphi_k(z)}{\omega - z} + \frac{\left(\Delta + \frac{k}{F+1}\right)\varphi_k(z)}{(\omega - z)^2}, \tag{23}$$

where $\Re$ denotes the radial ordering and $T(z)$ is the holomorphic part of the energy-momentum tensor:

$$T(z) = \sum_n \frac{L_n}{z^{n+2}}. \tag{24}$$

For the G_r a little more care is needed. One defines a χ-commutator as [49]

$$[A, B]_\chi := AB - \chi BA. \tag{25}$$

It is easy to see that

$$[A, BB']_{\chi\chi'} = [A, B]_\chi B' + \chi B[A, B']_{\chi'}. \tag{26}$$

Now, if we use

$$[G, \theta]_q = 0, \tag{27}$$

then the second equation of (14) leads to

$$[G_r, \varphi_k]_{q^{-k}} = z^{r+(1/(F+1))}q^{-k}(k+1)_{q^{-1}}\varphi_{k+1},$$
$$0 \le k \le F - 1, \tag{28}$$

$$[G_r, \varphi_F]_{q^{-F}} = \frac{q^{-F}}{(F)_{q^{-1}}!}\left[z^{r+(1/(F+1))}\partial_z\varphi_0 \right. \tag{29}$$
$$\left. + (F+1)\left(r + \frac{1}{F+1}\right)\Delta z^{r-(F/(F+1))}\varphi_0\right].$$

This can also be written in terms of the operator-product expansion. To do this, however, one should first define a

proper radial ordering for the supersymmetry generator and the component fields. Defining

$$\Re[S(w)\phi_k(z)] := \begin{cases} S(w)\phi_k(z), & |w| > |z|, \\ q^{-k}\phi_k(z)S(w), & |w| < |z|, \end{cases} \tag{30}$$

where

$$S(z) := \sum_r \frac{G_r}{z^{r+(F+2)/(F+1)}}, \tag{31}$$

one arrives at

$$\Re[S(\omega)\varphi_k(z)] \sim q^{-k}(k+1)_{q^{-1}}\frac{\varphi_{k+1}}{\omega - z},$$
$$0 \le k \le F - 1,$$
$$\Re[S(\omega)\varphi_F(z)] \sim \frac{q^{-F}}{(F)_{q^{-1}}!}\left[\frac{\partial_z\varphi_0}{\omega - z} + \frac{(F+1)\Delta\varphi_0}{(\omega - z)^2}\right]. \tag{32}$$

What we really use to restrict the correlation functions is that part of the algebra of which the central extension is trivial, that is, the algebra generated by $[L_{-1}, L_0, G_{-1/(F+1)}]$ and their antiholomorphic counterparts $[\bar{L}_{-1}, \bar{L}_0, \bar{G}_{-1/(F+1)}]$.

4 Two-point functions

The two point functions should be invariant under the action of the subalgebra generated by L_{-1}, L_0 and $G_{-1/(F+1)}$. This means that

$$\langle 0|[L_{-1}, \phi_k\phi'_{k'}]|0\rangle = 0, \tag{33}$$
$$\langle 0|[L_0, \phi_k\phi'_{k'}]|0\rangle = 0, \tag{34}$$
$$\langle 0|[G_{-1/(F+1)}, \phi_k\phi'_{k'}]_{q^{-k-k'}}|0\rangle = 0. \tag{35}$$

We have used the shorthand notation $\phi_k = \phi_k(z)$ and $\phi'_{k'} = \phi'_{k'}(z')$. ϕ and ϕ' are primary superfields of weight Δ and Δ', respectively. Equations (26) and (27) imply

$$\langle \phi_k\phi'_{k'}\rangle = \frac{A_{k,k'}}{(z - z')^{\Delta + \Delta' + (k+k')/(F+1)}}. \tag{36}$$

This is simply due to the fact that ϕ_k and $\phi'_{k'}$ are primary fields of the weight $\Delta + k/(F+1)$ and $\Delta' + k'/(F+1)$, respectively. Note that it is not required that these weights be equal to each other, since we have not included L_1 in the subalgebra.

Relation (28) relates A_{k_1,k'_1} with A_{k_2,k'_2}, if

$$k_1 + k'_1 - (k_2 + k'_2) = 0, \mod F + 1. \tag{37}$$

Therefore, there remain $F + 1$ independent constants in the $(F+1)^2$ correlation functions. So there are correlation functions of grade $0, 1, 2, \cdots$, and F. We have the following relations between the constants $A_{k,k'}$:

$$A_{k+1,k'} = -q^{-k'}\frac{1 - q^{-(k'+1)}}{1 - q^{-(k+1)}}A_{k,k'+1},$$
$$0 \le k, k' \le F - 1,$$

$$A_{k+1,F} = \frac{-q\left(\Delta + \Delta' + \frac{F+k}{F+1}\right)}{(F)_{q^{-1}}!(k+1)_{q^{-1}}} A_{k,0},$$
$$0 \le k \le F-1,$$
$$A_{F,k'+1} = \frac{q^{k'}\left(\Delta + \Delta' + \frac{F+k'}{F+1}\right)}{(F)_{q^{-1}}!(k'+1)_{q^{-1}}} A_{0,k'},$$
$$A_{F,0} = q^F A_{0,F}. \tag{38}$$

Using these, one can write (29) as

$$\langle \phi_k \phi'_{k'} \rangle = A_{k+k'} f_{k,k'}(z-z'), \tag{39}$$

where

$$A_{k+k'} := A_{k+k',0} = A_K. \tag{40}$$

So far, everything has been calculated for the chiral fields. But the generalization of this to full fields is not difficult. Following [44] and using exactly the same reasoning, it is seen that

$$\langle \varphi_{k\bar{k}}(z,\bar{z}) \varphi'_{k'\bar{k}'}(z',\bar{z}') \rangle = A_{K\bar{K}} q^{-k\bar{k}} f_{k,k'}(z-z') \bar{f}_{\bar{k},\bar{k}'}(\bar{z}-\bar{z}'). \tag{41}$$

Here $\bar{f}$ is the same as f with $\Delta \to \bar{\Delta}$ and $\Delta' \to \bar{\Delta}'$, and

$$K = k+k' \mod F+1, \quad \bar{K} = \bar{k}+\bar{k}' \mod F+1. \tag{42}$$

5 Logarithmic two point functions

Suppose that the first component field $\varphi_0(z)$ of the chiral superprimary field $\Phi(z,\theta)$ has a logarithmic counterpart $\varphi'_0(z)$:

$$[L_n, \varphi'_0(z)] = [z^{n+1}\partial_z + (n+1)z^n \Delta]\varphi'_0(z)$$
$$+ (n+1)z^n \varphi_0(z). \tag{43}$$

Following [39,45], one can show that $\varphi'_0(z)$ is the first component field of a new superfield $\phi'(z,\theta)$, which is the formal derivative of the superfield $\phi(z,\theta)$. One defines the fields $f'_r(z)$ through

$$[G_r, \varphi'_0(z)] =: z^{r+1/(F+1)} f'_r(z). \tag{44}$$

Then, by a reasoning similar to that presented in [39,45], that is acting on both sides by L_m and using the generalized Jacobi identity, it is seen that all the f_r are the same. Denoting this field by φ'_1, we have

$$[G_r, \varphi'_0(z)] =: z^{r+1/(F+1)} \varphi'_1(z). \tag{45}$$

One can continue and construct other component fields. It is found that

$$[G_r, \varphi'_k]_{q^{-k}} = z^{r+(1/(F+1))} q^{-k}(k+1)_{q^{-1}} \varphi'_{k+1},$$
$$0 \le k \le F-1,$$
$$[G_r, \varphi'_F]_{q^{-F}} = \frac{q^{-F}}{(F)_{q^{-1}}!} \left[z^{r+(1/(F+1))}\partial_z \varphi'_0 \right.$$
$$+ (F+1)\left(r+\frac{1}{F+1}\right)\Delta z^{r-(F/(F+1))}\varphi'_0$$
$$\left. + (F+1)\left(r+\frac{1}{F+1}\right) z^{r-(F/(F+1))}\varphi_0 \right], \tag{46}$$

and

$$[L_n, \varphi'_k] = z^{n+1}\partial_z \varphi'_k + (n+1)z^n\left(\Delta + \frac{k}{F+1}\right)\varphi'_k$$
$$+ (n+1)z^n \varphi_k. \tag{47}$$

Combining the primed component fields in the chiral superfield ϕ', one can write the action of the L_n and G_r on it as

$$[L_n, \phi'] = \left[z^{n+1}\partial_z + (n+1)z^n\left(\Delta + \frac{\Lambda}{F+1}\right)\right]\phi'$$
$$+ (n+1)z^n \phi, \tag{48}$$
$$[G_r, \phi'] = z^{r+(1/(F+1))}\left[\delta_\theta + \frac{\theta^F}{(F)_{q^{-1}}!}\partial_z\right]\phi'$$
$$+ \frac{F+1}{(F)_{q^{-1}}!}\left(r+\frac{1}{F+1}\right)z^{r-(F/(F+1))}\Delta\theta^F\phi'$$
$$+ \frac{F+1}{(F)_{q^{-1}}!}\left(r+(1/(F+1))\right)\theta^F\phi. \tag{49}$$

It is easy to see that these are formal derivatives of (14) with respect to Δ, provided one defines

$$\phi'(z,\theta) = \frac{\mathrm{d}\phi}{\mathrm{d}\Delta}. \tag{50}$$

The two superfields ϕ and ϕ' thus combine in a two-dimensional Jordanian block of quasi-primary fields. The generalization of the above results in an m-dimensional Jordanian block:

$$[L_n, \phi^{(i)}] = \left[z^{n+1}\partial_z + (n+1)z^n\left(\Delta + \frac{\Lambda}{F+1}\right)\right]\phi^{(i)}$$
$$+ (n+1)z^n\phi^{(i-1)}, \quad 1 \le i \le m-1, \tag{51}$$

and

$$[G_r, \phi^{(i)}] = z^{r+(1/(F+1))}\left[\delta_\theta + \frac{\theta^F}{(F)_{q^{-1}}!}\partial_z\right]\phi^{(i)}$$
$$+ \frac{F+1}{(F)_{q^{-1}}!}\left(r+\frac{1}{F+1}\right)z^{r-(F/(F+1))}\Delta\theta^F\phi^{(i)}$$
$$+ \frac{F+1}{(F)_{q^{-1}}!}\left(r+\frac{1}{F+1}\right)z^{r-(F/(F+1))}\theta^F\phi^{(i-1)},$$
$$1 \le i \le m-1. \tag{52}$$

One can regard these as formal derivatives of (14) with respect to Δ, provided one defines

$$\phi^{(i)} = \frac{1}{i!}\frac{\mathrm{d}^i\phi^{(0)}}{\mathrm{d}\Delta^i}. \tag{53}$$

It is then easy to see that

$$\langle \varphi^{(i)}_k \varphi'^{(j)}_{k'} \rangle = \frac{1}{i!}\frac{1}{j!}\frac{\mathrm{d}^i}{\mathrm{d}\Delta^i}\frac{\mathrm{d}^j}{\mathrm{d}\Delta'^j}\langle \varphi^{(0)}_k \varphi'^{(0)}_{k'} \rangle. \tag{54}$$

The correlator in the left-hand side is obtained from (39), and in differentiating with respect to the weights one

should regard the constants A_K as functions of the weights.

One can similarly define Jordanian blocks of full fields. This generalization is obvious:

$$\phi^{(ij)}(z, \bar{z}, \theta, \bar{\theta}) = \frac{1}{i!} \frac{1}{j!} \frac{\mathrm{d}^i}{\mathrm{d}\Delta^i} \frac{\mathrm{d}^j}{\mathrm{d}\bar{\Delta}^j} \phi^{(0)}(z, \bar{z}, \theta, \bar{\theta}). \quad (55)$$

The correlators of the component fields are then

$$\langle \varphi^{(ij)}_{k\bar{k}} \varphi^{(lm)}_{k'\bar{k}'} \rangle = \frac{1}{i!j!l!m!} \frac{\mathrm{d}^i}{\mathrm{d}\Delta^i} \frac{\mathrm{d}^j}{\mathrm{d}\bar{\Delta}^j} \frac{\mathrm{d}^l}{\mathrm{d}\Delta'^l} \frac{\mathrm{d}^m}{\mathrm{d}\bar{\Delta}'^m}$$
$$\times \langle \varphi^{(00)}_{k\bar{k}}(z, \bar{z}) \varphi'^{(00)}_{k'\bar{k}'}(z', \bar{z}') \rangle. \quad (56)$$

The correlator in the right-hand side is obtained from (41), and when differentiating with respect to the weights, the constants $A_{K\bar{K}}$ are regarded as functions of the weights.

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